Maxwell's Equations in Free Space and Plane Waves in Free Space

Maxwell's equations are the foundation of classical electromagnetism. They describe how electric and magnetic fields interact and propagate. In free space, where there are no charges ($\rho=0$) and no currents ($J=0$), these equations simplify significantly. Combined with the concept of plane waves, they provide the basis for understanding how electromagnetic waves travel through empty space.

Maxwell's Equations in Free Space

Maxwell's equations consist of four coupled differential equations:

1. Gauss's Law for Electric Fields

$\nabla \cdot E=0$

This equation states that the divergence of the electric field (E) in free space is zero. Since there are no charges in free space ($\rho=0$), the electric field lines neither originate nor terminate in free space.

2. Gauss's Law for Magnetic Fields

$\nabla \cdot B=0$

This equation implies that the divergence of the magnetic field (B) is always zero. In free space, this reflects the absence of magnetic monopoles, meaning magnetic field lines are always closed loops.

3. Faraday's Law of Electromagnetic Induction

$\nabla \times E=-\partial B/\partial t$

This equation describes how a time-varying magnetic field generates a circulating electric field. It explains phenomena such as electromagnetic induction.

4. Ampère-Maxwell Law

$\nabla \times B=\mu_0 \epsilon_0 \partial E/\partial t$

In free space, where no current density (J) exists, the curl of the magnetic field is proportional to the rate of change of the electric field. The constants μ_0 (permeability of free space) and ϵ_0 (permittivity of free space) appear here.